Eur. Phys. J. C (2016) 76:100
DOI 10.1140/epjc/s10052-016-3952-9

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Configurational entropy in $f(R, T)$ brane models

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Received: 3 September 2015 / Accepted: 17 February 2016 / Published online: 25 February 2016

Abstract In this work we investigate generalized theories of gravity in the so-called configurational entropy (CE) context. We show, by means of this information-theoretical measure, that a stricter bound on the parameter of $f(R, T)$ brane models arises from the CE. We find that these bounds are characterized by a valley region in the CE profile, where the entropy is minimal. We argue that the CE measure can play a new role and might be an important additional approach to selecting parameters in modified theories of gravitation.

1 Introduction

Although ΛCDM cosmological model provides a great match between theory and observation [1], which is why it usually is referred to as the "concordance model", a number of shortcomings, as the cosmological constant and hierarchy problems, still await convincing explanations. While by assuming general relativity as the gravitational theory, those problems cannot be solve straightforwardly, higher order derivative and extradimensional theories might contribute efficiently to solving these issues.

Note that $f(R)$ and $f(R, T)$ cosmological models [2–10], with R and T being, respectively, the Ricci scalar and the trace of the energy-momentum tensor, are able to describe the cosmic acceleration our universe is undergoing [11,12] with no need of invoking a cosmological constant. On the other hand, the hierarchy problem can be solved by assuming that our observable universe is a sub-manifold which is embedded in an anti-de Sitter five-dimensional space (AdS_5), called the bulk, as in the Randall–Sundrum braneworld model [13]. In fact, it has been shown in [14] that braneworld models can explain the cosmic acceleration as an effect of the leaking of gravity to the extra dimension.

In this work we will deal with braneworld models in the presence of scalar fields [15–18]. Departing from the original Randall and Sundrum proposal, which leads to a thin braneworld scenario, the coupling with scalar fields leads to thick braneworlds. The presence of a scalar field lets the warp function behave smoothly, yielding such a thickness. This possibility has opened a new area of study, and here we quote Refs. [19–26] for some work on the subject.

Specifically, we will consider the $f(R, T)$ gravity in such a thick braneworld scenario. Note that although $f(R)$ brane models have already been presented in the literature [27–35], due to its recent elaboration, $f(R, T)$ gravity still lacks a significant number of applications in the braneworld. Anyhow, it is a remarkable the fact that in [36], the authors have pioneered such an approach.

Here, rather than a cosmological approach, we will investigate $f(R, T)$ brane models from the configurational entropy (CE) perspective. Gleiser and Stamatopoulos (GS) have proposed in [37] such a new physical quantity, which brings about additional information as regards some parameters of a given model for which the energy density is localized. They have shown that the higher the energy that approximates the actual solution, the higher its relative CE, which is defined as the absolute difference between the actual function CE and the trial function CE. The CE is able to solve situations where the energies of the configurations are degenerate. In this case, it can be used to select the best system configuration.

Although it has been recently proposed, the CE has already been used to study the non-equilibrium dynamics of spontaneous symmetry breaking [38], to obtain the stability bound for compact objects [39], to investigate the emergence of localized objects during inflationary preheating [40], and to distinguish configurations with energy-degenerate spatial profiles [41]. Solitons, Lorentz symmetry breaking, supersymmetry, and entropy were employed using the CE concept [42–45]. The CE for traveling solitons reveals that the best value of the parameter responsible for breaking the Lorentz symmetry is that where the energy density is distributed

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equally around the origin. It was argued that the information-theoretical measure of traveling solitons in Lorentz symmetry violation scenarios can be very important to probe situations where the parameters responsible for breaking the symmetries are arbitrary, with the CE selecting the best value of the parameter in the model. Another interesting work about CE was presented in [46], where it is responsible for identifying the critical point in the context of continuous phase transitions. Finally, in braneworld scenarios it was shown that CE can be employed to demonstrate a high organizational degree in the structure of the system configuration for large values of a parameter of the sine-Gordon model [47]. The entropic information-measure in the context of $f(R)$ braneworlds with non-constant curvature has also been studied [48].

In this work, we calculate the CE in $f(R, T)$ brane models. We obtain its profile and reveal the information content of such models. The paper is organized as follows. In the next section we will review the concept of CE measure. In Sect. 3, we outline the basic theoretical structure for generalized $f(R, T)$ theories of gravity. In Sect. 4, we show two distinct models in $f(R, T)$ theory and its solutions. In Sect. 5, we describe the calculations of the CE for $f(R, T)$ theories. We also show the corresponding CE profile. Finally, in Sect. 6, we present our conclusions and directions for future work.

2 An overview of configurational entropy measure

Here, we will review the very recent work by GS [37], where it was shown that, in analogy with Shannon's information entropy, a CE measure in a functional space can be used to select the best fit solution with spatially localized energy. Another interesting consequence highlighted by GS is the fact that the CE relates the dynamical and informational content of physical models. In this case, this information-theoretical measure might extract information about the different solutions which is related to their spatial profiles. Thus, following [37], let us begin by writing the CE as

$$\sigma_c[f] = -\int \mathrm{d}^d\omega \tilde{f}(\omega) \ln[\tilde{f}(\omega)], \tag{1}$$

where d is the number of spatial dimensions, $\tilde{f}(\omega) = f(\omega)/f_{\max}(\omega)$, where $f_{\max}(\omega)$ is the maximal modal fraction, that is, the mode giving the highest contribution. In this case, the function $f(\omega)$, which was defined as the modal fraction, is represented as

$$f(\omega) = \frac{|\mathcal{F}(\omega)|^2}{\int \mathrm{d}^d\omega\, |\mathcal{F}(\omega)|^2}. \tag{2}$$

The function $\mathcal{F}(\omega)$ represents the Fourier transform of the energy density of the configuration. It is important to remark that the energy density must be square-integrable since in such cases the entropy can be well defined.

We will extend the procedure presented in [37], which is absolutely general when applied to systems with spatially localized energy for a scalar field theory that describes generalized gravity coupled to a scalar field in five-dimensional space-time. Following that work, it is possible to obtain the entropy of the configurations assuming that $\mathcal{F}(\omega)$ obeys the Fourier transform:

$$\mathcal{F}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathrm{d}x \mathrm{e}^{i\omega y} \rho(y). \tag{3}$$

We are working with spatially localized energy densities, which are part of a set of square-integrable bounded functions $\rho(y) \in L^2(\mathbf{R})$ and their Fourier transforms $\mathcal{F}(\omega)$. It is also important to remark that the Plancherel theorem states that

$$\int_{-\infty}^{\infty} \mathrm{d}y\, |\rho(y)|^2 = \int_{-\infty}^{\infty} \mathrm{d}\omega\, |\mathcal{F}(\omega)|^2\,. \tag{4}$$

As argued by GS, the information-entropic measure approach might take into account the dynamical and the informational contents of models with localized energy configurations. In this case, the CE provides a complementary perspective to situations where strictly energy-based arguments are inconclusive. In fact, as pointed out in [37], higher CE correlates with higher energy, and consequently the configuration is more disordered. Therefore, the information-theoretical measure is responsible for indicating which solution is the most ordered one among a family of infinite solutions.

Thus, we will apply this new approach to investigate generalized gravity theories coupled to a real scalar field. As we will see, important consequences will arise from the CE concept. Furthermore, we will show that the CE provides a stricter bound on the parameters of the $f(R, T)$ models.

3 $f(R, T)$ theories of gravity

In this section, we present a brief review of $f(R, T)$ theories of gravity. In this context, by Harko et al. the seminal work entitled $f(R, T)$ gravity [5] was presented some years ago. In that context the authors showed that it is possible to construct a modified theory of gravity, where the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar R and of the trace of the energy-momentum tensor T. From a physical viewpoint, the dependence on T may originate with the presence of exotic imperfect fluids or quantum effects. Therefore, as shown in [5], in $(3+1)$ dimensions the action S for the $f(R, T)$ modified theories of gravity can be written in the form

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\,f(R,T) + \int d^4x\sqrt{-g}\,\mathcal{L}_m, \tag{5}$$

where $f(R,T)$ is an arbitrary function of R and T, g is the determinant of the metric $g_{\mu\nu}$ with μ, ν assuming the values 0, 1, 2, 3, and $\mathcal{L}_m$ is the matter Lagrangian density. Note that we use the natural system of units with $G = c = 1$, so that the Einstein gravitational constant is defined as $\kappa^2 = 8\pi$.

We emphasize that the $f(R,T)$ modified theory of gravity has been analyzed in the literature in several scenarios. For instance, we can find studies in cosmological models [6–10,49], in quantum chromodynamics [50], in a Bianchi Type-II string with magnetic field [51], and in thermodynamics [52–55].

On the other hand, recently, Bazeia et al., motivated by Refs. [5,56], have found thick-brane models in $f(R,T)$ generalized theories of gravity in an AdS_5 warped geometry with an extra dimension of infinite extent [36]. It has been shown that two distinct choices of the functional form of $f(R,T)$ induce quantitative modifications in the thick-brane profile, without changing its qualitative behavior. Another interesting result obtained in [36] is the fact that the gravity sector remains linearly stable in each choice of $f(R,T)$ analyzed.

The theory used by Bazeia et al. for describing generalized gravity coupled to a scalar field in five-dimensional space-time, with an extra dimension y of infinite extent, is given by the following action:

$$S = \int d^4x\,dy\sqrt{|g|}\left[-\frac{1}{4}f(R,T) + \mathcal{L}_s\right], \tag{6}$$

where $\mathcal{L}_s$ is the Lagrange density, $g = \det(g_{ab})$, with the indices a and b running from 0 to 4, the signature of the metric is $(+----)$ and we use units such that $4\pi G^{(5)} = 1$. In this case, the Lagrangian density is put into the form

$$\mathcal{L}_s = \frac{1}{2}\nabla_a\phi\nabla^a\phi - V(\phi). \tag{7}$$

From the equation above, the energy-momentum tensor of the scalar field in this theory is given by

$$T_{ab} = -\frac{1}{2}g_{ab}\nabla_c\phi\nabla^c\phi + g_{ab}V + \nabla_a\phi\nabla_b\phi. \tag{8}$$

In addition, the corresponding equation of motion for the scalar field and the modified Einstein equation can be written as

$$\nabla_a\nabla^a\phi + \frac{3}{4}\nabla_a(f_T\nabla^a\phi) + \frac{5}{4}V_\phi f_T + V_\phi = 0, \tag{9}$$

$$f_R R_{ab} - \frac{1}{2}g_{ab}f + (g_{ab}\Box - \nabla_a\nabla_b)f_R = 2T_{ab} + \frac{3}{2}f_T\nabla_a\phi\nabla_b\phi, \tag{10}$$

where $V_\phi \equiv dV/d\phi$, $f_R \equiv df/dR$, and $f_T \equiv df/dT$.

Following the approach presented in [36], we will work with the metric

$$ds^2 = e^{2A}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2, \tag{11}$$

where y is the extra dimension, $\eta_{\mu\nu}$ is the usual Minkowski metric in the four-dimensional space-time with signature $(+,-,-,-)$, and e^{2A} is the so-called warp factor. In particular, when the warp function A and the field ϕ are static and depend only on the extra dimension y, the equation of motion and the modified Einstein equations become

$$\left(1+\frac{3}{4}f_T\right)\phi'' + \left[(4+3f_T)A' + \frac{3}{4}f_T'\right]\phi' = \left(1+\frac{5}{4}f_T\right)V_\phi, \tag{12}$$

$$\frac{2}{3}\phi'^2\left(1+\frac{3}{4}f_T\right) = -A''f_R + \frac{1}{3}A'f_R' - \frac{1}{3}f_R'', \tag{13a}$$

$$V(\phi) - \frac{\phi'^2}{2}\left(1+\frac{3}{2}f_T\right) = 2(A'^2 + A'')f_R - \frac{f}{4} - 2A'f_R', \tag{13b}$$

where the prime denote derivatives with respect to the extra dimension.

Here, it is important to highlight that this particular approach, where we consider the static case with $A = A(y)$ and $\phi = \phi(y)$, makes possible the finding of innovative analytical solutions for the theory. A good example are the interesting solutions found in [36]. In that work, two distinct models for $f(R,T)$ theories of gravity have been studied, and it was shown that the gravity sector of the thick braneworld configurations remains linearly stable. Moreover, each model induces new quantitative modifications in the thick-brane profile. Thus, in order to make the present work more comprehensive and didactical, in the next section we will present those two examples and their solutions.

4 Two specific models and their solutions

In the present section, we consider two particular classes of $f(R,T)$ modified gravity models, which were investigated in [36]. In that reference, it has been found that in such particular cases, both the field ϕ and the warp function A can be obtained analytically.

4.1 Case 1: $f(R,T) = R - \alpha T^n$

In the first case, the $f(R,T)$ function will be given by [5,8,36,57]

$$f(R,T) = R - \alpha T^n, \tag{14}$$

where α and n are real parameters. In this case, when $n = 1$ the exact solutions presented in [36] for the field ϕ and the warp function A are written as

$$\phi(y) = \frac{1}{b}\arcsin[\tanh(By)], \tag{15}$$

$$A(y) = \frac{2\gamma a}{3B}\ \ln[\mathrm{sech}(By)], \tag{16}$$

where γ, a, and b are real parameters, and $B \equiv 4\gamma a b^2/(4-3\alpha)$. Furthermore, the energy density can be put in the form

$$\rho(y) = \frac{16(\gamma a)^2}{3(4-5\alpha)}\left\{-1+\left[1+\frac{3B(1-\alpha)}{\gamma a(4-3\alpha)}\right]\mathrm{sech}^2(By)\right\} \times[\mathrm{sech}(By)]^{\frac{4\gamma a}{3B}}. \tag{17}$$

4.2 Case 2: $f(R,T) = R+\beta R^2-\alpha T$

Now, let us investigate another model, where $f(R,T)$ is represented by [58–60]

$$f(R,T) = R+\beta R^2-\alpha T, \tag{18}$$

with α and β being real parameters. In this case the solutions for the field and warp function can be written in the following form:

$$\phi(y) = \pm\sqrt{\frac{6}{4-3\alpha}-\frac{320\beta k^2}{4-3\alpha}}\,\mathcal{E}\left(\Phi,\Xi\right), \tag{19}$$

$$A(y) = \ln[\mathrm{sech}(ky)], \tag{20}$$

where k is a positive parameter and $\mathcal{E}(\Phi,\Xi)$ is the elliptic integral of the second kind with

$$\Phi\equiv\phi_s = \arcsin\Big[\tanh(ky)\Big],\ \text{and}\ \Xi\equiv\frac{392\beta k^2}{160\beta k^2-3}. \tag{21}$$

Here, the energy density is given by

$$\rho(y) = -\frac{12k^2}{4-5\alpha}\tilde{S}^2+\frac{12k^2(6-5\alpha)}{(4-3\alpha)(4-5\alpha)}\tilde{S}^4-\frac{8\beta k^4}{4-5\alpha}\tilde{S}^2 \times\left[10-\frac{116(6-5\alpha)}{4-3\alpha}\tilde{S}^2+\frac{98(8-7\alpha)}{4-3\alpha}\tilde{S}^4\right], \tag{22}$$

where $\tilde{S} = \mathrm{sech}(ky)$.

5 CE in $f(R,T)$ theories

In this section, we will study the entropic profile of the generalized gravity theories here presented.

5.1 CE for $f(R,T) = R-\alpha T^n$

For the first case, we can rewrite the energy density (17) in the following form:

$$\rho(y) = \sum_{j=1}^{2} A_j \mathrm{sech}^{\delta_j}(By), \tag{23}$$

where $\delta_1\equiv\delta$ and $\delta_2\equiv\delta+2$, with $\delta\equiv 4\gamma a/B$. Furthermore, we have

$$A_1 \equiv -\frac{16(\gamma a)^2}{3(4-5\alpha)}, \tag{24}$$

$$A_2 \equiv -\frac{16(\gamma a)^2}{3(4-5\alpha)}\left[1+\frac{3B(1-\alpha)}{\gamma a(4-3\alpha)}\right]. \tag{25}$$

Therefore substituting the energy density given by Eq. (23) into Eq. (3), we can obtain, after some arduous calculations, the following Fourier transform:

$$\mathcal{F}(\omega) = \sum_{j=1}^{2}\sum_{\ell=1}^{2} A_{j,\ell}\,{}_2\mathcal{H}_1[\delta_j,\mu_{j,\ell};\mu_{j,\ell}+1;-1], \tag{26}$$

where ${}_2\mathcal{H}_1[\delta_j,\mu_{j,\ell};\mu_{j,\ell}+1;-1]$ is the hypergeometric function, and we use the definitions

$$\mu_{j,\ell}\equiv[B\delta_j-(-1)^{\ell+1}i\omega]/2B, \tag{27}$$

$$A_{j,\ell}\equiv\frac{2^{\delta_j-1/2}A_j[B\delta_j+(-1)^{\ell+1}i\omega]}{\sqrt{\pi}(B^2\delta_j^2+\omega^2)}. \tag{28}$$

Moreover, we have

$$\int_{-\infty}^{\infty}\mathrm{d}\omega\,|\mathcal{F}(\omega)|^2 = \frac{1}{B}\sum_{j=1}^{2}\sum_{\ell=1}^{2}\frac{2^{\delta_{j,\ell}-1}\Im_{j,\ell}\Gamma^2(\delta_{j,\ell}/2)}{\Gamma(\delta_{j,\ell})}, \tag{29}$$

with

$$\delta_{j,\ell}\equiv\delta_j+\delta_\ell,\ \Im_{j,\ell}\equiv A_jA_\ell. \tag{30}$$

Now, we can write the modal fraction in the form

$$f(\omega) = B\left[\frac{\sum_{j,\ell,m,n=1}^{2}A_{j,\ell}A_{m,n}^{*}\,{}_2\mathcal{H}_1^{(j,\ell)}\,{}_2\mathcal{H}_1^{(m,n)*}}{\sum_{j,\ell=1}^{2}\frac{2^{\delta_{j,\ell}-1}\Im_{j,\ell}\Gamma^2(\delta_{j,\ell}/2)}{\Gamma(\delta_{j,\ell})}}\right], \tag{31}$$

where

$${}_2\mathcal{H}_1^{(j,\ell)} = {}_2\mathcal{H}_1[\delta_j,\mu_{j,\ell};\mu_{j,\ell}+1;-1], \tag{32}$$

$${}_2\mathcal{H}_1^{(m,n)} = {}_2\mathcal{H}_1[\delta_m,\mu_{m,n};\mu_{m,n}+1;-1]. \tag{33}$$

Figure 1 depicts $f(\omega)$ for different values of α. As can be seen, the modal fraction profile is influenced by the α

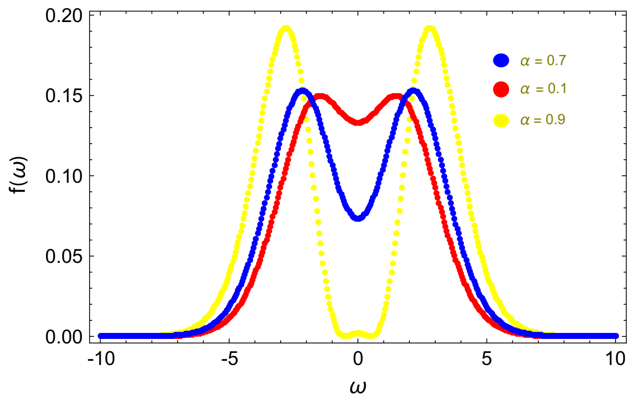

Fig. 1 Modal fractions for different values of α

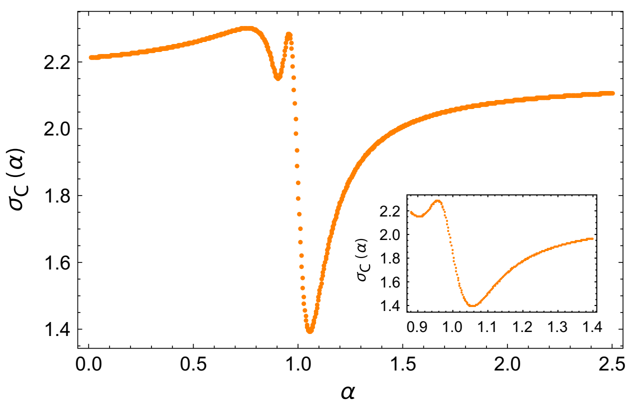

Fig. 2 Configurational entropy for $f(R, T) = R - \alpha T^n$

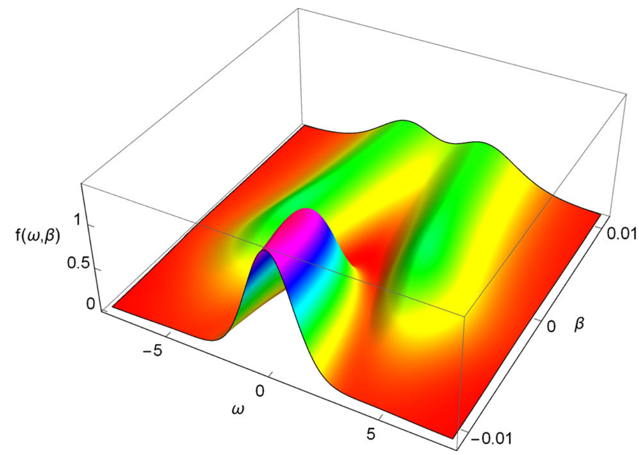

Fig. 3 Modal fraction with $\alpha = 0.1$

parameters. In this case, we can check that the α parameter is responsible for the appearance of a split in the modal fraction, where there are two peaks which are symmetric with respect to the $\omega = 0$ axis.

The CE for $f(R, T) = R - \alpha T^n$ is plotted in Fig. 2 below.

5.2 CE for $f(R, T) = R + \beta R^2 - \alpha T$

Here, the energy density can be put in the form

$$\rho(y) = \sum_{j=1}^{3} \tilde{A}_j \mathrm{sech}^{\tilde{\delta}_j}(ky), \tag{34}$$

where $\tilde{\delta}_1 = 2$, $\tilde{\delta}_2 = 4$, and $\tilde{\delta}_3 = 6$, with

$$\tilde{A}_1 \equiv -\frac{4k^2(3 + 20\beta k^2)}{4 - 5\alpha}, \tag{35}$$

$$\tilde{A}_2 \equiv \frac{4k^2(6 - 5\alpha)(3 + 232\beta k^2)}{(4 - 5\alpha)(4 - 3\alpha)}, \tag{36}$$

$$\tilde{A}_3 \equiv -\frac{784\beta k^4(8 - 7\alpha)}{(4 - 5\alpha)(4 - 3\alpha)}. \tag{37}$$

From the energy density (34) we can find the Fourier transform

$$\mathcal{F}(\omega) = \sum_{j=1}^{3}\sum_{\ell=1}^{3} \tilde{A}_{j,\ell}\ {}_2\mathcal{G}_1[\tilde{\delta}_j, \tilde{\mu}_{j,\ell}; \tilde{\mu}_{j,\ell} + 1; -1], \tag{38}$$

where we use the following definitions:

$$\tilde{\mu}_{j,\ell} \equiv [k\tilde{\delta}_j - (-1)^{\ell+1} i\omega]/2k, \tag{39}$$

$$\tilde{A}_{j,\ell} \equiv \frac{2^{\tilde{\delta}_j - 1/2}\tilde{A}_j[k\tilde{\delta}_j + (-1)^{\ell+1} i\omega]}{\sqrt{\pi}(k^2\tilde{\delta}_j^2 + \omega^2)}. \tag{40}$$

Thus, we can find that

$$\int_{-\infty}^{\infty} \mathrm{d}\omega\, |\mathcal{F}(\omega)|^2 = \frac{1}{k}\sum_{j=1}^{3}\sum_{\ell=1}^{3} \frac{2^{\tilde{\delta}_{j,\ell}-1}\wp_{j,\ell}\Gamma^2(\delta_{j,\ell}/2)}{\Gamma(\delta_{j,\ell})}, \tag{41}$$

with

$$\wp_{j,\ell} \equiv \tilde{A}_j \tilde{A}_\ell. \tag{42}$$

Therefore, we can write

$$f(\omega) = k\left[\frac{\sum_{j,\ell,m,n=1}^{3} \tilde{A}_{j,\ell}\tilde{A}^*_{m,n}\ {}_2\mathcal{G}_1^{(j,\ell)}\ {}_2\mathcal{G}_1^{(m,n)*}}{\sum_{j,\ell=1}^{3} \frac{2^{\tilde{\delta}_{j,\ell}-1}\wp_{j,\ell}\Gamma^2(\tilde{\delta}_{j,\ell}/2)}{\Gamma(\tilde{\delta}_{j,\ell})}}\right], \tag{43}$$

where

$${}_2\mathcal{G}_1^{(j,\ell)} = {}_2\mathcal{G}_1[\tilde{\delta}_j, \tilde{\mu}_{j,\ell}; \tilde{\mu}_{j,\ell} + 1; -1], \tag{44}$$

$${}_2\mathcal{G}_1^{(m,n)} = {}_2\mathcal{G}_1[\tilde{\delta}_m, \tilde{\mu}_{m,n}; \tilde{\mu}_{m,n} + 1; -1]. \tag{45}$$

Figures 3 and 4 below show the modal fraction for $\alpha = 0.1$ and $\alpha = 0.98$, respectively. From that figures, we can note that the α parameters induce the formation of two peaks which are symmetric in relation to the $\omega = 0$ axis.

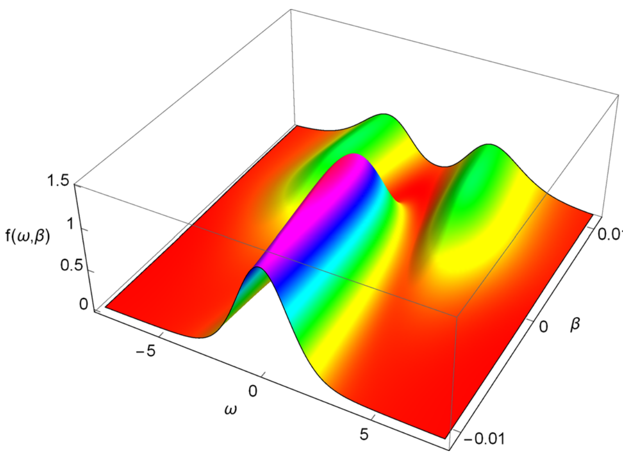

Fig. 4 Modal fraction with $\alpha = 0.98$

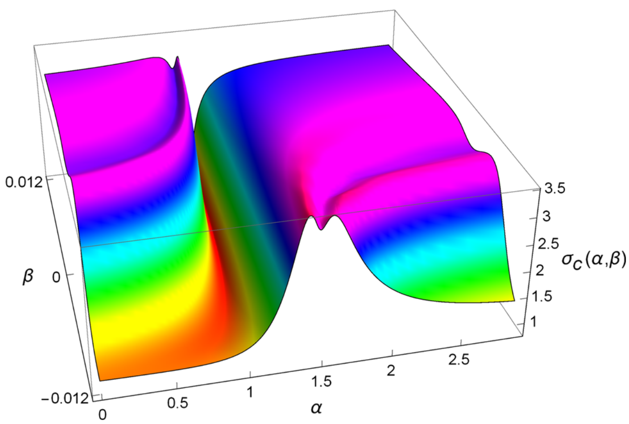

Fig. 5 Configurational entropy for $f(R,T) = R + \beta R^2 - \alpha T$

In Fig. 5 below we plot the CE for $f(R,T) = R + \beta R^2 - \alpha T$.

6 Conclusions

In this work we investigated $f(R,T)$ braneworld models in the CE context. We showed, by means of this information-theoretical measure, that a stricter bound on the parameter of $f(R,T)$ theories of gravity arises from the CE. We found that these bounds are characterized by a valley region in the CE profile, where the entropy is minimal.

From inspection of such results, we can see that in the case where $f(R,T) = R - \alpha T^n$, the CE shows a rich structure for varying α. In this case, there is a sharp minimum at the value $\alpha \simeq 1.07$, where the corresponding CE is given by $\sigma_c \simeq 1.4$. This result leads us to conclude that the CE can be used in order to extract rich information as regards the structure of the configurations, which is clearly related to their profiles. Therefore, we found that the best ordering for the solutions are those given by $\alpha \simeq 1.07$. At this point it is important to remark that the idea of using configurational entropy as a selection of modified gravity models has some potentially interesting features. In an effective theory with an infinite number of unknown parameters, it seems impossible to identify the right configuration. However, there is a way of evading such a problem, setting a cutoff scale [14,61,62]. Thus, we will select solutions that are below the cutoff scale. Assuming that the trace of energy-momentum tensor has [energy]/[volume] dimensionality, it will have m^5 dimensionality (in five dimensions). In order for αT^n to have the same dimensionality of the scalar curvature R it is necessary that α has m^{2-5n} dimensionality, for some mass m. Therefore, it can be concluded that the cutoff mass is given by

$$m_{\text{cutoff}} = (M_5 m^{2-5n})^{1/(5-5n)}, \tag{46}$$

where M_5 is the five-dimensional Planck mass. Thus, we can see that m_{cutoff} is smaller than m if $n > 1$. For example, if $n = 2$, we have $\alpha = 1/m^8$ and the cutoff mass is $m_{\text{cutoff}} = (M_5 m^{-8})^{-1/5} = (M_5\alpha)^{-1/5}$.

Moreover, when $f(R,T) = R + \beta R^2 - \alpha T$, the minimal CE corresponds to a valley region where the best values for α and β are localized in the bottom of the valley. In this way, the information-theoretical measure of generalized theories of gravity, such as $f(R,T)$ theories, opens a new window to probe situations where the parameters responsible for the control of the theory are, at first sight, arbitrary. In this case, the CE selects the best values.

Due to its recent elaboration, $f(R,T)$ gravity still lacks a considerable number of applications in braneworlds. Because of that, for now it is impossible to compare the constraints obtained for the parameters of $f(R,T)$ theory, here, via the CE approach, with those which can be obtained from observational cosmology.

$f(R,T)$ gravity, indeed, has been applied to extradimensional models [6,8,63–66], however, those are derived from Kaluza–Klein theory [67–69]. In other words, those works consider compactified extra dimensions. On the other hand, the present work considers the existence of a space-like extra dimension of infinite extent.

As we have quoted above, Bazeia et al. have pioneered the $f(R,T)$ braneworld study in [36]. As argued by Bazeia et al., the parameter α in Eq. (14), which had, here, its value constrained by the CE approach, controls the thickness of the brane solutions and narrows the warp factors as it increases. The favorable value for α derived above allows for obtaining the thin-brane Randall–Sundrum limit when $y = \pm\infty$ according to [36]. On the other hand, the presence of the term R^2 in (18) generates the splitting of the brane, as in [27]. Therefore, according to Eq. (18), the bounds on the values of β obtained via CE may dictate the consequences of such an interesting effect.

Finally, we can conclude that the CE provides a complementary perspective to investigate generalized theories of gravity such as Gauss–Bonnet, Weyl, and Brans–Dicke theories, as well as several other modified theories of gravitation. Another interesting line of investigation in which CE can play an important rule is the evolution of the domain walls in Euclidean space [70]. In this case, the information-theoretical measure can be used as a discriminant to understand the phase transition processes.

Acknowledgments RACC thanks to UFABC and CAPES for financial support, and PHRSM thanks to FAPESP for financial support. We also would like to thank Denis Dalmazi for helpful discussions and for valuable remarks. RACC and PHRSM also would like to thank the anonymous referee for the valuable comments.

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